Equilibria and Incentives in Private Information Economies[☆]

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Abstract

This paper considers three solution concepts in a large private information economy, namely, Walrasian expectations equilibrium, private core, and insurance equilibrium. It shows that these three concepts coincide with each other when the agents are informationally negligible in such an economy. In contrast to the finite-agent setting, one can construct a large private information economy in which incentive compatibility fails completely in the sense that almost every agent in any Walrasian expectations equilibrium/private core/insurance equilibrium allocation has the incentive to misreport her type.

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1. Introduction

It is well-known that in a finite-agent private information economy, it is in general not possible to write contracts that are incentive compatible, individually rational and Pareto efficient. On the other hand, one may hope that such an inconsistency problem disappears in a large market where the informational

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influence of a single agent could be negligible. In particular, McLean and Postlewaite (2002, 2003, 2005) used a notion of "informational smallness" and showed the existence of incentive compatible, approximate Walrasian equilibria and an approximate core equivalence result via countable independent replicas of a fixed private information economy with finitely many agents.[1] Sun and Yannelis (2007a,b, 2008) obtained some exact results in private information economies with a continuum of agents, where the private information is negligible in the sense that the independent private signals of individual agents can influence only a negligible group of other agents.[2]

The above results have been achieved under the assumptions that agents' private types are purely informational (*i.e.*, the utility functions and endowments are type-irrelevant) and the agents have contingent consumptions based on the type profiles for all the agents.[3] A natural question then arises: what will occur if the agents' types are allowed to enter the utility functions and endowments, and each agent's consumption is contingent on her private type? In this paper, we study the relevant issues systematically. Besides the usual solution concepts of Walrasian equilibrium and core in such a setting, the informational negligibility also leads us to consider insurance equilibrium which allows complete insurance of individual-level risks.[4] Under the assumption that the private types of a continuum of agents are independent, we establish the equivalence of the above three solution concepts in Proposition 1.[5] It is clear that those three notions may not be equivalent to each other in the finite-agent setting.

As regarding the issue of incentive compatibility, a subtle difference between finite-agent and continuum-agent economies has been discussed in the first paragraph. In general, one would expect incentive compatibility to hold in a large economy even though it may fail in the corresponding finite-agent case. However, we will obtain a result contrary to such an intuition in the set-up as considered in this paper, namely, the agents' private types influence the utility functions, endowments and consumptions. More specifically, in a finite-agent private information economy with private information measurable endowments, it is easy to see that the combination of the exact feasibility and the private measurability on allocations leads to type-independent net trades, which implies the incentive compatibility.[6] It is surprising that there exists

[1]For other related works, see, for example, Hammond (1979), Palfrey and Srivastava (1986), Gul and Postlewaite (1992), Mas-Colell and Vives (1993), Krasa and Shafer (2001) and McLean and Postlewaite (2004). See Debreu and Scarf (1963) for the classical approach based on countable replicas of a finite-agent economy.

[2]Besides showing the existence of incentive compatible Walrasian equilibria and an core equivalence result in Sun and Yannelis (2007a,b), Sun and Yannelis (2008) provided a stronger consistence result: every *ex ante* efficient allocation is incentive compatible under some stronger conditions.

[3]In the sequel, we shall use "type" interchangeably with "signal".

[4]The notions of Walrasian equilibrium and core as considered in our setting are Walrasian expectations equilibrium and private core. These two solution concepts were introduced in Radner (1968) and Yannelis (1991) respectively, where agents' consumptions are contingent on their private information. Malinvaud (1972) and Sun (2006) considered the possibility of insurance via cross-state income transfers by individual agents in finite-agent economies and large economies respectively. To guarantee full insurance in large economies with only idiosyncratic risk, they worked with constant-price equilibria, namely, insurance equilibria in their setting.

[5]Einy *et al.* (2001) showed the equivalence of Walrasian equilibrium and private core, where the space of information for all the agents has only finitely many points. It means that there is a finite partition of the atomless agent space so that the agents in each partition have exactly the same private information; consequently, the relevant signal process cannot be independent across agents. In contrast, a key feature in the models as considered in McLean and Postlewaite (2002, 2003, 2005), Sun and Yannelis (2007a,b, 2008), and this paper is to allow the agents to have idiosyncratic information. In particular, the information partition of agent i in this paper is $\big\{\{t_i\} \times T_{-i} \mid t_i \in T^0\big\}$. It then allows for the application of the exact law of large numbers under the assumption of independent signals.

[6]See Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994) for the discussion of various incentive compatibility

a *large* private information economy in which incentive compatibility fails completely in the sense that almost every agent in any Walrasian expectations equilibrium/private core/insurance equilibrium allocation has the incentive to misreport her type; see Proposition 2. The basic intuition behind this result is that unlike the finite-agent setting, the exact feasibility requirement no longer imposes major restrictions on the privately measurable allocations in large economies so that the private information of an individual agent has an effect on her net trade (via the type dependency of her endowment and utility function). Thus, (almost) any individual can misreport her private information and become better off.

This paper is organized as follows. Section 2 introduces a model of private information economies with a continuum of agents. Section 3 then states the definitions of Walrasian expectations equilibrium, private core, and insurance equilibrium. The main results are presented in Section 4 while the proofs are given in Section 6.

2. Private information economy

Let an atomless probability space[7] $(I, \mathcal{I}, \lambda)$ be the space of economic agents. Let a finite set $T^0 = \{q_1, q_2, \ldots, q_L\}$ be the space of all the possible signals/types for individual agents (its power set denoted by $\mathcal{T}^0$), and $(T, \mathcal{T}, \mathbf{P})$ a probability space that models the private signal profiles for all the agents, where T is the space of functions from I to T^0.[8] Indeed, the probability space $(T, \mathcal{T}, \mathbf{P})$, which captures the uncertainty on the signals for all the agents, can be regarded as the space of states. Given a private signal profile $t \in T$, for each $i \in I$, $t(i)$ (also denoted by t_i) is the private signal of agent i while t_{-i} is the restriction of the signal profile t to the set $I \setminus \{i\}$ of agents different from i; let T_{-i} be the set of all such t_{-i}. For simplicity, we shall assume that $(T, \mathcal{T})$ has a product structure so that T is the product of T^0 and T_{-i}, while $\mathcal{T}$ is the product σ-algebra of $\mathcal{T}^0$ and a σ-algebra $\mathcal{T}_{-i}$ on T_{-i}. For any $t \in T$ and $t'_i \in T^0$, we shall adopt the usual notation (t'_i, t_{-i}) to denote the signal profile whose value is t'_i for agent i, and the same as t for other agents.

Let f be a private signal process from $I \times T$ to T^0 such that $f(i,t) = t_i$ for any $(i,t) \in I \times T$. To capture the idea of idiosyncratic information, we need to impose the independence condition on the private signal process f, which would in general lead to the non-measurability of f with respect to the usual product σ-algebra. Thus, we need to work with an extension of the usual product space $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes \mathbf{P})$ that retains the Fubini property. Such an extension, denoted by $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P})$, is called a Fubini extension.[9] Its formal definition is given in Section 6. Throughout this paper, we assume that the private signal process f is $\mathcal{I} \boxtimes \mathcal{T}$-measurable.

For each $i \in I$, we define agent i's private signal distribution μ_i as follows. For each $q \in T^0$, $\mu_i(\{q\}) = \mathbf{P} f_i^{-1}(\{q\})$ is the probability of agent i receiving the private signal q. For notational simplicity, $\mu_i(\{q\})$ is

issues of private core in the more general coalitional setting.

[7] We use the convention that all probability spaces are countably additive. A probability space $(I, \mathcal{I}, \lambda)$ (or its σ-algebra) is atomless if for any non-negligible subset $E \in \mathcal{I}$, there is an $\mathcal{I}$-measurable subset E' of E such that $0 < \lambda(E') < \lambda(E)$.

[8] In the literature, one usually assumes that different agents have possibly different sets of signals and requires that the agents take all their own signals with positive probability. For notational simplicity, we choose to work with a common set T^0 of signals, but allow zero probability for some of the redundant signals. There is no loss of generality in this latter approach.

[9] See Sun (2006) for some discussion of the measurability problem and definition of a Fubini extension.

often abbreviated as $\mu_i(q)$. Let $T_i^0 = \{q \in T^0 \mid \mu_i(q) > 0\}$ be the set of all private signals that matter to agent i (in the probabilistic sense).

We consider a large economy with private information. In this economy, when a state $t \in T$ occurs, each agent $i \in I$ is informed with her private signal $t_i \in T^0$. The common commodity space is the positive orthant $\mathbb{R}_+^m$. The utility function of each agent depends on her consumption $z = (z_1, z_2, \ldots, z_m) \in \mathbb{R}_+^m$ and the private signal $q \in T^0$ she receives, where z_j is the quantity of the j-th commodity in the consumption z. Thus, we can let u be a mapping from $I \times \mathbb{R}_+^m \times T^0$ to $\mathbb{R}_+$ such that for each $i \in I$, $u(i, z, q)$ (also denoted by $u_i(z, q)$) is the utility of agent i at the consumption $z \in \mathbb{R}_+^m$ and the private signal $q \in T^0$. For any fixed $z \in \mathbb{R}_+^m$ and $q \in T^0$, $u(\cdot, z, q)$ is assumed to be $\mathcal{I}$-measurable; for any fixed $i \in I$ and $q \in T^0$, $u(i, \cdot, q)$ is assumed to be continuous, concave and strictly monotone.[10] The endowment of each agent depends on the private signal $q \in T^0$ she receives. That is, we can let e be a mapping from $I \times T^0$ to $\mathbb{R}_+^m$ with $e(i, q)$ as the initial endowment of agent i when her private signal is $q \in T^0$. For any fixed $q \in T^0$, $e(\cdot, q)$ is assumed to be an integrable mapping from $(I, \mathcal{I}, \lambda)$ to $\mathbb{R}_+^m$. For $\mathbf{P}$-almost all $t \in T$, we assume that $\int_I e\big(i, f(i,t)\big)\, \mathrm{d}\lambda(i) \in \mathbb{R}_{++}^m$.[11] The collection

$$\mathcal{E} = \langle (I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P}), u, e, f \rangle$$

is called a *private information economy*.

3. Basic definitions

In this section, we consider three solution concepts for private information economies: Walrasian expectations equilibrium, private core, and insurance equilibrium. Let $\mathcal{E} = \langle (I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P}), u, e, f \rangle$ be a private information economy.

- A *consumption plan* y for individual agents is an integrable mapping from $(T, \mathcal{T}, \mathbf{P})$ to $\mathbb{R}_+^m$. For each state $t \in T$, $y(t) \in \mathbb{R}_+^m$ is the consumption when t occurs. Let $L^1(\mathbf{P}, \mathbb{R}_+^m)$ denote the set of consumption plans.

- If agent i's consumption plan is y, then her (*ex ante*) expected utility is given as

$$U_i(y) = \int_T u_i\big(y(t), f(i,t)\big)\, \mathrm{d}\mathbf{P}(t).$$[12]

  When agent i's consumption plan y is contingent on her private signal, her expected utility becomes

$$U_i(y) = \int_T u_i\Big(y\big(f(i,t)\big), f(i,t)\Big)\, \mathrm{d}\mathbf{P}(t) = \sum_{q \in T^0} \mu_i(q) \cdot u_i\big(y(q), q\big).$$

[10] This means that if $z, z' \in \mathbb{R}_+^m$, $z \geq z'$ with $z \neq z'$, then $u(i, z, q) > u(i, z', q)$.

[11] A vector is said to be in $\mathbb{R}_{++}^m$ if and only if all its components are positive.

[12] Since $u(i, \cdot, q)$ is concave for any fixed i and q, there are constants c and d in $\mathbb{R}_+$ such that $u(i, z, q) \leq c\|z\| + d$ for any $z \in \mathbb{R}_+^m$, where $\|\cdot\|$ is the Euclidean norm. From this condition, it is clear that $\int_T u_i\big(y(i,t), f(i,t)\big)\, \mathrm{d}\mathbf{P}(t)$ is finite.

- A *price* p is a measurable mapping from $(T, \mathcal{T}, \mathbf{P})$ to Δ_m, where Δ_m denotes the unit simplex in $\mathbb{R}^m_+$. For each state $t \in T$, $p(t)$ denotes the commodity price vector when t occurs.
- An *allocation* x is an integrable mapping from $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P})$ to $\mathbb{R}^m_+$. For each $(i,t) \in I \times T$, $x(i,t)$ (also denoted by $x_i(t)$) is interpreted as agent i's consumption at state t. When agents' consumption depends only on their private signals, we simply write it as $x(i,q)$ (also denoted by $x_i(q)$) for $q \in T^0$. In this case, x is viewed as a mapping from $I \times T^0$ to $\mathbb{R}^m_+$.
- Given a price p, agent i's *budget set* is defined as

$$B_i(p) = \left\{ y \in L^1(\mathbf{P}, \mathbb{R}^m_+) \;\middle|\; \int_T p(t) \cdot y(t) \, \mathrm{d}\mathbf{P}(t) \leq \int_T p(t) \cdot e\big(i, f(i,t)\big) \, \mathrm{d}\mathbf{P}(t) \right\}.$$

Definition 1 (Walrasian expectations equilibrium)**.** A *Walrasian expectations equilibrium* is a pair of an allocation x^* and a price p^* such that the following three conditions hold.

(1) For each $i \in I$, x_i^* depends only on agent i's private signal.

(2) x^* is feasible, *i.e.*, $\int_I x^*\big(i, f(i,t)\big) \, \mathrm{d}\lambda(i) = \int_I e\big(i, f(i,t)\big) \, \mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$.

(3) For λ-almost all $i \in I$, x_i^* maximizes $U_i(\cdot)$ subject to $B_i(p^*)$.

An allocation x^* is said to be a *Walrasian expectations equilibrium allocation* if there is a price p^* such that (x^*, p^*) forms a Walrasian expectations equilibrium.

In Definition 1, Condition (1) indicates that each agent's consumption plan is contingent on her private signal. Condition (2) is the standard state-wise market clearing condition. Condition (3) says that each agent makes a consumption plan such that the plan maximizes her expected utility subject to the budget set.

In a private information economy, agents in a coalition could exchange information for making their consumption plans. Several formulations of core have thus been proposed depending on the amount of information to be shared in a coalition. In most applications, agents do not have an incentive to reveal their own private information (think of situations of moral hazard or adverse selection). Hence, there could be no information sharing among agents, and each agent may only use her own private signal in making consumption plans. This idea is formulated as the private core, initiated by Yannelis (1991). Note that in a private core allocation, each agent is limited to her own private information. Indeed as it was shown in Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994), any private information measurable efficient allocation is incentive compatible, but sharing information results to non-incentive compatible allocations.

In a private information economy $\mathcal{E}$, let x and x' be two allocations, and W a coalition.[13] The allocation x' is said to block the allocation x on W if

(1) $\int_W x'\big(i,t\big) \, \mathrm{d}\lambda(i) = \int_W e\big(i, f(i,t)\big) \, \mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$,

(2) $U_i(x_i') > U_i(x_i)$ for λ-almost all $i \in W$.

[13] A coalition is a measurable subset of agents with positive measure under λ.

When each agent's consumption plan in the allocation x' depends only on her private signal, Condition (1) becomes:

(1') $\int_W x'\big(i, f(i,t)\big)\, \mathrm{d}\lambda(i) = \int_W e\big(i, f(i,t)\big)\, \mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$.

Since $u(i, \cdot, q)$ is continuous and strictly monotone for any fixed $i \in I$ and $q \in T^0$, Condition (2) is equivalent to:

(2') $U_i(x'_i) \geq U_i(x_i)$ for λ-almost all $i \in W$ and $\lambda\big(\{i \in W \mid U_i(x'_i) > U_i(x_i)\}\big) > 0$.

We state the definition of private core below.

Definition 2 (Private core)**.** The *private core* of $\mathcal{E}$ is the set of all the allocations x^* such that the following three conditions hold.

(1) For each $i \in I$, x^*_i depends only on agent i's private signal.

(2) x^* is feasible, *i.e.*, $\int_I x^*\big(i, f(i,t)\big)\, \mathrm{d}\lambda(i) = \int_I e\big(i, f(i,t)\big)\, \mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$.

(3) There is no coalition W and no allocation x which depends only on the corresponding agents' private signal such that x blocks x^* on W.

Condition (1) requires that each agent can only use her own private information for making consumption plans. Condition (2) implies that the markets are cleared for almost every state $t \in T$. Condition (3) says that no coalition of agents (while each agent in the coalition is limited to her own private information for making consumption plans) can redistribute their initial endowments among themselves for almost every state t and make the expected utility of each agent in the coalition better off.

The third solution concept we consider is insurance equilibrium. This notion is used to study insurance systems where each agent takes on individual risks, and makes choices of consumption to spread risks across states. When there is a large number of risk-bearing agents in a market and no collective risk prevails, it is often conjectured that contingent commodity prices are the multiple of "sure prices" and an objective probability. The insurance model was studied by Malinvaud (1972) and Sun (2006). In particular, Malinvaud (1972) considered individual risks in the context of Pareto-optimal allocations for a sequence of finite markets with given finitely many types and each type having N agents (*i.e.*, a replica economy) with convex, continuous and complete preorderings, and stochastically independent risks across replicas. Some versions of the classical law of large numbers were used to claim that the budget of the insurance system would be approximately balanced in the absence of transaction costs when N goes to infinity. To have a system of full insurance, Sun (2006) studied a general model for a large economy with individual risks, and showed that the risks are insurable if and only if they are essentially pairwise independent.

To capture the feature of full insurance, we will work with insurance equilibria whose price systems are constant as in Condition (1) of the following definition.

Definition 3 (Insurance equilibrium)**.** An *insurance equilibrium* is a pair of an allocation x^* and a price p^* such that the following four conditions hold.

(1) The price p^* is constant, *i.e.*, $p^*(t) = p^*(t')$ for any t and t' in T.

(2) For each $i \in I$, x^*_i depends only on agent i's private signal.

(3) x^* is feasible, *i.e.*, $\int_I x^*\big(i, f(i,t)\big)\,\mathrm{d}\lambda(i) = \int_I e\big(i, f(i,t)\big)\,\mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$.

(4) For λ-almost all $i \in I$, x_i^* maximizes $U_i(\cdot)$ subject to $B_i(p^*)$.

An allocation x^* is said to be an *insurance equilibrium allocation* if there is a price p^* such that (x^*, p^*) forms an insurance equilibrium.

In Definition 3, Condition (2) indicates that each agent's consumption is contingent on her private signal. Condition (3) is the standard state-wise market clearing condition. Condition (4) requires that each agent makes consumption plans to maximize her expected utility subject to the budget constraint. When the price is constant, we notice that each agent i's budget set becomes

$$B_i(p) = \left\{ y \in L^1(\mathbf{P}, \mathbb{R}^m_+) \;\middle|\; p \cdot \int_T y(t)\,\mathrm{d}\mathbf{P}(t) \le p \cdot \int_T e\big(i, f(i,t)\big)\,\mathrm{d}\mathbf{P}(t) \right\}.$$

4. The results

Before providing the main results, we first state a definition which formalizes the intuition of idiosyncratic information.

Definition 4 (Idiosyncratic signal process)**.** *The signal process f is called an* idiosyncratic signal process *if it is essentially pairwise independent in the sense that for λ-almost all $i \in I$, the random variables f_i and f_j from $(T, \mathcal{T}, \mathbf{P})$ to T^0 are independent for λ-almost all $j \in I$.*

We establish the equivalence among Walrasian expectations equilibrium, private core and insurance equilibrium in the following.

Proposition 1. *Let $\mathcal{E}$ be a private information economy. Assume that f is an idiosyncratic signal process. Then the following statements are equivalent:*

(1) *x^* is a Walrasian expectations equilibrium allocation of $\mathcal{E}$.*

(2) *x^* is a private core allocation of $\mathcal{E}$.*

(3) *x^* is an insurance equilibrium allocation of $\mathcal{E}$.*

In the following, we study the incentive compatibility for the three solution concepts considered above. In a finite-agent private information economy with private information measurable endowments, the exact feasibility and the private measurability on allocations lead to type-independent net trades for each agent, which implies the incentive compatibility. However, we will illustrate that the incentive compatibility does not hold any more in large private information economies.

For each $i \in I$, denote $\mathbf{P}_{t_i}^{T_{-i}}$ the conditional probability measure on the space $(T_{-i}, \mathcal{T}_{-i})$ when agent i's signal is $t_i \in T^0$.

Definition 5. For an allocation x, an agent $i \in I$, and private signals $t_i, t_i' \in T^0$, let

$$U_i(x_i, t_i' \mid t_i) = \int_{T_{-i}} u_i\big(e_i(t_i) + x_i(t_i', t_{-i}) - e_i(t_i'), t_i\big)\,\mathrm{d}\mathbf{P}_{t_i}^{T_{-i}}(t_{-i})$$

be the expected utility of agent i when she receives the private signal t_i but misreports as t_i', where the corresponding net trade $x_i(t_i', t_{-i}) - e_i(t_i')$ is feasible in the sense that $e_i(t_i) + x_i(t_i', t_{-i}) - e_i(t_i')$ is in the consumption space $\mathbb{R}_+^m$.[14] The allocation x is said to be incentive compatible if for λ-almost all $i \in I$,

$$U_i(x_i, t_i \mid t_i) \geq U_i(x_i, t_i' \mid t_i)$$

holds for all the signals $t_i, t_i' \in T_i^0$ of agent i with the corresponding net trades being feasible.

The following proposition shows that there exists a large private information economy in which almost every agent in any Walrasian expectations equilibrium/private core/insurance equilibrium allocation has the incentive to misreport her type.

Proposition 2. *There exists a private information economics $\mathcal{E} = \langle (I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P}), u, e, f \rangle$ such that (1) it satisfies all the conditions on the information structure in Section 2, and f is an idiosyncratic signal process; (2) in any Walrasian expectations equilibrium/private core/insurance equilibrium allocation, almost every agent has the incentive to misreport her type.*

In fact, the one-good large economy as constructed for proof of Proposition 2 in Subsection 6.4 shows that those agents with the expected endowments below a critical value have the incentive to misreport for a higher income while the other agents have the incentive to misreport for a lower income. The basic intuition is that because the exact feasibility requirement does not impose any substantive restriction on the privately measurable allocations, the private information of an individual agent has different effects on her net trade, depending on her expected income level. In particular, (1) for an agent with her expected endowment below a critical value, she can misreport a low endowment to a high endowment so that she can get a higher net trade due to the higher payoff effect of a reported high endowment; (2) for an agent with her expected endowment above the critical value, she can misreport a high endowment to a low endowment so that she can get a higher net trade due to the higher endowment effect of a reported low endowment.[15]

5. Concluding remarks

Since reality suggests that the uncertainty in an economy with many agents comes from both macro and micro levels, one may work with the assumption that the private types of a continuum of agents are conditionally independent, given the macro level shocks.[16] Our set-up and results (without macro states)

[14]When a deviation to a given allocation is considered, one needs to impose a feasibility assumption on the relevant net trade; we thank an anonymous referee for suggesting this point. Roberts and Postlewaite (1976, p. 117) required the summation of a net trade in $S(p)$ and the endowment of an agent to be in her consumption set. The model in Hammond (1979, Section 2) started with a feasible set of net trades for each agent directly. When the accessibility to a public good was considered, Hellwig (2007) modeled the consumption set to be $\{0, 1\}$ and worked with a feasible net trade allocation whose summation with the initial allocation to be in $\{0, 1\}$ (see page 521).

[15]As mentioned in the introduction, when the agents' utility functions and endowments are type-irrelevant and the agents have contingent consumptions based on the type profiles for all the agents, Sun and Yannelis (2008) showed (together with other strong conditions) that every *ex ante* efficient allocation is incentive compatible. All those conditions (including the type irrelevancy conditions on the utility functions and on the endowments) are also shown to be minimal via some examples.

[16]It follows from Theorem 1 of Hammand and Sun (2008) that the conditional independence assumption is generally satisfied.

can be generalized in a trivial way to the case with finitely many macro states, where (1) the agents observe the macro state and their own private signals, (2) the relevant objects (utilities, endowments, and consumptions) depend on macro states and private signals, and (3) the private signal process is essentially pairwise independent conditional on macro states. However, in the models of McLean and Postlewaite (2002, 2003, 2005) and Sun and Yannelis (2007a,b, 2008), the macro states are not observable, and the consumptions are contingent on the private signals profiles for all the agents.

In this paper, we consider Walrasian expectations equilibrium, private core, and insurance equilibrium in large private information economies. Informational negligibility (idiosyncratic information) implies their existence and the equivalence among them. In contrast to finite-agent economies, we construct a large private information economy where almost every agent in any Walrasian expectations equilibrium/private core/insurance equilibrium allocation has the incentive to misreport her type. In view of recent work on equilibrium theory under ambiguity,[17] one may explore the possibility whether the conflict between the efficiency and the incentive compatibility as in Proposition 2 could be resolved in such a setting.

6. Proofs

To prove Propositions 1–2, we begin the analysis by stating the framework of Fubini extension and the exact law of large numbers in Subsection 6.1 and induced large economies in Subsection 6.2, which will be useful in the derivation of the main results.

6.1. Fubini extension and the exact law of large numbers

For the convenience of the reader, here we state the definition of a Fubini extension and the exact law of large numbers as in Sun (2006). Let probability spaces $(I, \mathcal{I}, \lambda)$ and $(T, \mathcal{T}, \mathbf{P})$ be the index and sample spaces, respectively. Let $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes \mathbf{P})$ be the usual product probability space. Given a function f on $I \times T$ (not necessarily $\mathcal{I} \otimes \mathcal{T}$-measurable), for any $(i, t) \in I \times T$, let f_i be the function $f(i, \cdot)$ on T and f_t the function $f(\cdot, t)$ on I. A formal definition of the Fubini extension is given below.

Definition 6. A probability space $(I \times T, \mathcal{W}, \mathbf{Q})$ extending the usual product space $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes \mathbf{P})$ is said to be a Fubini extension of $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes \mathbf{P})$ if for any real-valued $\mathbf{Q}$-integrable function f on $(I \times T, \mathcal{W})$,

(1) the two functions f_i and f_t are integrable, respectively, on $(T, \mathcal{T}, \mathbf{P})$ for λ-almost all $i \in I$, and on $(I, \mathcal{I}, \lambda)$ for $\mathbf{P}$-almost all $t \in T$;

(2) $\int_T f_i \, \mathrm{d}\mathbf{P}$ and $\int_I f_t \, \mathrm{d}\lambda$ are integrable, respectively, on $(I, \mathcal{I}, \lambda)$ and $(T, \mathcal{T}, \mathbf{P})$, with $\int_{I \times T} f \, \mathrm{d}\mathbf{Q} = \int_I \left(\int_T f_i \, \mathrm{d}\mathbf{P} \right) \mathrm{d}\lambda = \int_T \left(\int_I f_t \, \mathrm{d}\lambda \right) \mathrm{d}\mathbf{P}$.

[17] de Castro *et al.* (2011) and de Castro and Yannelis (2013) among others applied the maximin expected utility to an asymmetric information economy with a finite number of states of nature. With the maximin expected utilities, agents take into account the worst possible state that can occur and choose the best possible allocations. de Castro and Yannelis (2013) showed that every efficient allocation is incentive compatible if and only if all individuals have maximin preferences. He and Yannelis (2015) considered the maximin expectations equilibrium and maximin core in a non-free disposal economy with countably many states of nature. The ambiguous economy modeling allowed them to obtain the existence of maximin expectations equilibrium and the consistency between the incentive compatibility and the efficiency. For recent developments see Angelopoulos and Koutsougeras (2015) and Liu (2016).

To reflect the fact that the probability space $(I \times T, \mathcal{W}, \mathbf{Q})$ has $(I, \mathcal{I}, \lambda)$ and $(T, \mathcal{T}, \mathbf{P})$ as its marginal spaces, as required by the Fubini property, it will be denoted by $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P})$.

The following is an exact law of large numbers for a continuum of independent random variables shown in Sun (2006).

Lemma 1. *Assume that a signal process f from $I \times T$ to T^0 is essentially pairwise independent in the sense that for λ-almost all $i \in I$, the random variables f_i and f_j from $(T, \mathcal{T}, \mathbf{P})$ to T^0 are independent for λ-almost all $j \in I$. Then the cross-sectional distribution λf_t^{-1} of the sample function f_t is the same as the distribution $(\lambda \boxtimes \mathbf{P}) f^{-1}$ of the process f viewed as a random variable on $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P})$ for $\mathbf{P}$-almost all $t \in T$.*

6.2. Induced large economy

From a large private information economy $\langle (I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P}), u, e, f \rangle$, we can construct an auxiliary large economy $\bar{\mathcal{E}}$. This economy and several properties will be used in the proofs of our main results.

For each $i \in I$, we define a function E_i from $(\mathbb{R}_+^m)^{T_i^0}$ to $\mathbb{R}_+^m$ as

$$E_i(y) = \sum_{q \in T_i^0} \mu_i(q) \cdot y(q) = \int_{T_i^0} y(q) \, \mathrm{d}\mu_i(q)$$

for any $y \in (\mathbb{R}_+^m)^{T_i^0}$. Note that if y' is a function from T^0 to $\mathbb{R}_+^m$ and its restriction on T_i^0 is y, then

$$\int_{T^0} y'(q) \, \mathrm{d}\mu_i(q) = \sum_{q \in T^0} \mu_i(q) \cdot y'(q) = \sum_{q \in T_i^0} \mu_i(q) \cdot y(q) = E_i(y),$$

where the second equality holds since $T_i^0 = \{q \in T^0 \mid \mu_i(q) > 0\}$. It means that $E_i(y)$ is equal to the expectation of y' with respect to μ_i. For this reason, $E_i(y')$ will also be used to denote the expectation of y' on the probability measure space $(T^0, \mathcal{T}^0, \mu_i)$. Moreover, E_i^{-1} is a correspondence from $\mathbb{R}_+^m$ to $(\mathbb{R}_+^m)^{T_i^0}$ so that $E_i^{-1}(z) = \{y \in (\mathbb{R}_+^m)^{T_i^0} \mid E_i(y) = z\}$ for each $z \in \mathbb{R}_+^m$.

For each $i \in I$, recall that $U_i(y) = \sum_{q \in T_i^0} \mu_i(q) \cdot u_i\big(y(q), q\big)$ for any $y \in (\mathbb{R}_+^m)^{T_i^0}$. We define a function $\bar{u}$ from $I \times \mathbb{R}_+^m$ to $\mathbb{R}_+$ by letting

$$\bar{u}(i, z) = \max_{y \in E_i^{-1}(z)} U_i(y)$$

for any $i \in I$ and $z \in \mathbb{R}_+^m$. For each $i \in I$, let $\bar{u}_i$ be the function $\bar{u}(i, \cdot)$ on $\mathbb{R}_+^m$. The following lemma shows that each $\bar{u}_i$ indeed defines a utility function.

Lemma 2. *For each $i \in I$, $\bar{u}_i$ is well-defined and continuous. Furthermore, if $u_i(z, q)$ is (strictly) monotone in z for each fixed $q \in T^0$, then $\bar{u}_i$ is (strictly) monotone.*

This lemma is proved in Sun (2006) (see Lemma 3.2 therein) for the case of complete and continuous preferences. An alternative proof that deals directly with utility functions is provided in the online appendix.

In the large economy $\bar{\mathcal{E}}$, each agent i's initial endowment is

$$\bar{e}(i) = E_i(e_i),$$

and her utility function is $\bar{u}(i,z)$. We summarize the *induced large economy* as

$$\bar{\mathcal{E}} = \langle (I, \mathcal{I}, \lambda), \bar{u}, \bar{e} \rangle.$$

The following two lemmas provide the connections between the relevant concepts in a private information economy and its induced large economy, which are important for proving the main results. The proofs of these two lemmas are given in the online appendix.

Lemma 3. *Let $\mathcal{E}$ be a private information economy and $\bar{\mathcal{E}}$ the induced large economy. Assume that f is an idiosyncratic signal process. If $\bar{x}$ is an allocation of $\bar{\mathcal{E}}$, then there exists an allocation x of $\mathcal{E}$ with the following properties:*

(i) *For each $i \in I$, x_i depends only on agent i's private signal;*

(ii) *For λ-almost all $i \in I$, $\bar{x}_i = E_i(x_i)$;*

(iii) *For λ-almost all $i \in I$, $\bar{u}_i(\bar{x}_i) = U_i(x_i)$;*

(iv) *For any coalition $W \in \mathcal{I}$, if $\int_W \bar{x}(i)\,\mathrm{d}\lambda(i) = \int_W \bar{e}(i)\,\mathrm{d}\lambda(i)$, then $\int_W x(i, f(i,t))\,\mathrm{d}\lambda(i) = \int_W e(i, f(i,t))\,\mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$. In particular, if $\bar{x}$ is feasible in $\bar{\mathcal{E}}$, then x is feasible in $\mathcal{E}$.*

Lemma 4. *Assume that f is an idiosyncratic signal process.*

(i) *Suppose that x is a private core allocation of $\mathcal{E}$. Define an allocation $\bar{x}$ for the economy $\bar{\mathcal{E}}$ such that $\bar{x}(i) = E_i(x_i)$ for each $i \in I$. Then $\bar{x}$ is a core allocation of $\bar{\mathcal{E}}$.*

(ii) *Suppose that $\bar{x}$ is a core allocation of $\bar{\mathcal{E}}$. Then there exists a private core allocation x of $\mathcal{E}$ such that $\bar{x}_i = E_i(x_i)$ and $\bar{u}_i(\bar{x}_i) = U_i(x_i)$ for λ-almost all $i \in I$.*

Lemma 4 shows that a private core allocation of an economy $\mathcal{E}$ can be mapped naturally to a core allocation of the induced large economy $\bar{\mathcal{E}}$ by taking the expectation of the private core allocation. Conversely, for a core allocation of $\bar{\mathcal{E}}$, there exists a private core allocation of $\mathcal{E}$ whose expectation is the core allocation. By the standard results in Aumann (1964, 1966), there exists a core allocation in $\bar{\mathcal{E}}$, which implies that the private core of $\mathcal{E}$ is also nonempty. Furthermore, each agent has the same level of utility (expected utility for agents in $\mathcal{E}$) in the two economies with these two allocations.

6.3. Proof of Proposition 1

We divide the proof into three parts.

Part 1: "(1) $\Rightarrow$ (2)". We prove it by contradiction. Suppose that x^* is not a private core allocation. Then there exist a coalition W and a feasible allocation x' such that $\int_W x'(i, f(i,t))\,\mathrm{d}\lambda(i) = \int_W e(i, f(i,t))\,\mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$ and $U_i(x'_i) > U_i(x^*_i)$ for λ-almost all $i \in W$. Since $U_i(x'_i) > U_i(x^*_i)$ and x^*_i maximizes $U_i(\cdot)$ subject to the budget $B_i(p^*)$, we have $x'_i \notin B_i(p^*)$ for λ-almost all $i \in W$. It follows that

$$\int_T p^*(t) \cdot x'\left(i, f(i,t)\right) \mathrm{d}\mathbf{P}(t) > \int_T p^*(t) \cdot e\left(i, f(i,t)\right) \mathrm{d}\mathbf{P}(t)$$

for λ-almost all $i \in W$.

Integrating the above inequality with respect to i on W, we get

$$\int_W \int_T p^*(t) \cdot x'\left(i, f(i,t)\right) \mathrm{d}\mathbf{P}(t) \, \mathrm{d}\lambda(i) > \int_W \int_T p^*(t) \cdot e\left(i, f(i,t)\right) \mathrm{d}\mathbf{P}(t) \, \mathrm{d}\lambda(i).$$

Since f is essentially pairwise independent, the exact law of large numbers as stated in Lemma 1 implies

$$\int_W p^*(t) \cdot x'\left(i, f(i,t)\right) \mathrm{d}\lambda(i) > \int_W p^*(t) \cdot e\left(i, f(i,t)\right) \mathrm{d}\lambda(i)$$

for $\mathbf{P}$-almost all $t \in T$.

On the other hand, $\int_W x'\left(i, f(i,t)\right) \mathrm{d}\lambda(i) = \int_W e\left(i, f(i,t)\right) \mathrm{d}\lambda(i)$ for $\mathbf{P}$-almost all $t \in T$ implies

$$\int_W p^*(t) \cdot x'\left(i, f(i,t)\right) \mathrm{d}\lambda(i) = \int_W p^*(t) \cdot e\left(i, f(i,t)\right) \mathrm{d}\lambda(i)$$

for $\mathbf{P}$-almost all $t \in T$. It is a contradiction.

Therefore, x^* is a private core allocation.

Part 2: "(2) $\Rightarrow$ (3)". We shall find a price p^* such that (x^*, p^*) is an insurance equilibrium.

By Lemma 4, we can construct a core allocation $\bar{x}^*$ for the induced large economy $\bar{\mathcal{E}}$ by letting $\bar{x}_i^* = E_i(x_i^*)$ for each $i \in I$. The standard Core Equivalence Theorem (see Aumann (1964)) indicates that there is a price $\bar{p}^*$ such that $(\bar{x}^*, \bar{p}^*)$ is a Walrasian equilibrium for the economy $\bar{\mathcal{E}}$. That is, $\bar{x}^*$ is a feasible allocation and $\bar{x}_i^*$ maximizes $\bar{u}_i(z)$ subject to $\bar{p}^* \cdot z \leq \bar{p}^* \cdot \bar{e}(i)$ for λ-almost all $i \in I$.

We will show that (x^*, p^*) is an insurance equilibrium for $\mathcal{E}$, where $p^*(t) = \bar{p}^*$ for each $t \in T$. Since x^* is a private core allocation, it is easy to see that (x^*, p^*) satisfies the first three conditions in the definition of insurance equilibrium (see Definition 3). It is left to show that for λ-almost all $i \in I$, x_i^* maximizes agent i's expected utility $U_i(\cdot)$ subject to her budget set $B_i(p^*)$.

To prove this, we first show that $\bar{u}_i(\bar{x}_i^*) = U_i(x_i^*)$ for λ-almost all $i \in I$. By Lemma 3, there exists a feasible allocation x' such that $U_i(x_i') = \bar{u}_i(\bar{x}_i^*)$ and $\bar{x}_i^* = E_i(x_i')$ for λ-almost all $i \in I$. If $U_i(x_i') = \bar{u}_i(\bar{x}_i^*) > U_i(x_i^*)$ holds on a subset of agents with positive measure under λ, then x^* is blocked by x' on this subset, contradicting the fact that x^* is a private core allocation. Hence, $U_i(x_i') = \bar{u}_i(\bar{x}_i^*) = U_i(x_i^*)$ for λ-almost all $i \in I$.

For any $y \in B_i(p^*)$, we have $p^* \cdot E_i(y) \leq p^* \cdot \bar{e}(i)$. Since $\bar{x}_i^*$ maximizes $\bar{u}_i(z)$ subject to $p^* \cdot z \leq p^* \cdot \bar{e}(i)$, we have $\bar{u}_i(E_i(y)) \leq \bar{u}_i(\bar{x}_i^*)$. On the other hand, $\bar{u}_i(E_i(y)) = \max_{y' \in E_i^{-1}(E_i(y))} U_i(y') \geq U_i(y)$, it follows immediately that $U_i(x_i^*) = \bar{u}_i(\bar{x}_i^*) \geq \bar{u}_i(E_i(y)) \geq U_i(y)$. That is to say, x_i^* maximizes $U_i(\cdot)$ subject to the budget set $B_i(p^*)$. Hence, (x^*, p^*) is an insurance equilibrium for $\mathcal{E}$.

Part 3: "(3) $\Rightarrow$ (1)". Routine. □

6.4. Proof of Proposition 2

We consider a one-good economy $\langle (I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P}), u, e, f \rangle$ as follows:

- The space of agents is an atomless probability space $(I, \mathcal{I}, \lambda)$.

- The space of individual types T^0 is $\{0,1\}$, and the space of type profiles is a probability space $(T, \mathcal{T}, \mathbf{P})$.
- Let the private signal process be a measurable mapping f from $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes \mathbf{P})$ to T^0 with $f(i,t) = t_i$ for any $i \in I$ and $t \in T$. Assume that f is essentially pairwise independent and $\mathbf{P} f_i^{-1}(\{0\}) = \frac{1}{2}$ for each $i \in I$.[18]
- The utility function is $u(i, z, t_i) = (1 + t_i) \ln(1 + t_i + z)$ for any $i \in I$ and $t_i \in T^0$.
- The endowment is $e(i, t_i) = \frac{1}{4} + c(i) \cdot t_i$ for any $i \in I$ and $t_i \in T^0$, where c is a function from I to $[0,1]$ which induces the uniform distribution on $[0,1]$.[19] For simplicity, $c(i)$ is usually written as c_i for each $i \in I$. Furthermore, the exact law of large numbers in Lemma 1 implies

$$\int_I e(i,t_i)\,\mathrm{d}\lambda(i) = \int_I \int_T \left(\tfrac{1}{4} + c(i) \cdot t_i\right) \mathrm{d}\mathbf{P}(t)\,\mathrm{d}\lambda(i) = \int_I \left(\tfrac{1}{4} + \tfrac{1}{2} c(i)\right) \mathrm{d}\lambda(i) = \tfrac{1}{2}$$

  for $\mathbf{P}$-almost all $t \in T$.

By the equivalence result in Proposition 1, we can simply consider Walrasian expectations equilibrium allocations. Let x be any Walrasian expectations equilibrium allocation. For each $i \in I$, x_i depends only on agent i's private signal t_i. Let $a_i = \int_T x_i(t_i)\,\mathrm{d}\mathbf{P}(t) = \frac{1}{2} x_i(0) + \frac{1}{2} x_i(1)$. Then Jensen's inequality implies that

$$U_i(x_i) = \tfrac{1}{2} \ln\left(1 + x_i(0)\right) + \ln\left(2 + x_i(1)\right) \leq \tfrac{1}{2} \ln\left(1 + \tfrac{2}{3} a_i\right) + \ln\left(1 + \tfrac{4}{3} a_i\right)$$

with equality only when $x_i(0) = \frac{2}{3} a_i$ and $x_i(1) = \frac{4}{3} a_i$. Since x is a Walrasian expectations equilibrium allocation, we have $x_i(0) + x_i(1) = e_i(0) + e_i(1)$ for λ-almost all $i \in I$. Thus, $2a_i = \frac{1}{2} + c_i$ for λ-almost all $i \in I$.

Define an allocation y by letting $y(i, t_i) = \frac{2}{3}(1 + t_i) a_i$. That is, $y(i,0) = \frac{2}{3} a_i$ and $y(i,1) = \frac{4}{3} a_i$. Thus, for λ-almost all $i \in I$, $U_i(x_i) \leq U_i(y_i)$ with equality only when $x_i = y_i$. By the exact law of large numbers in Lemma 1, we have

$$\int_I y(i,t_i)\,\mathrm{d}\lambda(i) = \int_I \int_T \tfrac{2}{3}(1 + t_i) a_i\,\mathrm{d}\mathbf{P}(t)\,\mathrm{d}\lambda(i) = \int_I \tfrac{2}{3}\left(1 + \tfrac{1}{2}\right) a_i\,\mathrm{d}\lambda(i) = \int_I a_i\,\mathrm{d}\lambda(i) = \tfrac{1}{2},$$

for $\mathbf{P}$-almost all $t \in T$. Thus, $\int_I y(i,t_i)\,\mathrm{d}\lambda = \int_I e(i,t_i)\,\mathrm{d}\lambda(i)$. That is, y is feasible.

Since x is also a private core allocation by Proposition 1, there is no coalition W and no allocation x' which depends only on private signal such that x' blocks x on W. Thus, there exists a subset A of I with $\lambda(A) = 1$ such that for any $i \in A$, $x_i(0) = y_i(0)$ and $x_i(1) = y_i(1)$. Let $B = \{i \in A \mid c_i < \frac{1}{4}\}$. For each $i \in B$, since $2a_i = \frac{1}{2} + c_i$, we have $\frac{2}{3} a_i < \frac{4}{3} a_i - c_i$. For any agent $i \in B$, we have

$$\begin{aligned} U_i(x_i, 1 \mid 0) &= \ln\left(1 + x_i(1) + e_i(0) - e_i(1)\right) = \ln(1 + \tfrac{4}{3} a_i - c_i) \\ &> \ln(1 + \tfrac{2}{3} a_i) = \ln\left(1 + x_i(0)\right) = U_i(x_i, 0 \mid 0). \end{aligned}$$

[18] Proposition 5.6 in Sun (2006) ensures the existence of such a process f.

[19] Since $(I, \mathcal{I}, \lambda)$ is atomless, such a function c exists.

We also have $e_i(0) + x_i(1) - e_i(1) = \frac{4}{3}a_i - c_i > \frac{2}{3}a_i \geq 0$; that is, the net trade $x_i(1) - e_i(1)$ is feasible.

Let $C = \{i \in A \mid c_i > \frac{1}{4}\}$. For each $i \in C$, we have $\frac{2}{3}a_i > \frac{4}{3}a_i - c_i$, and hence

$$\begin{aligned} U_i(x_i, 0 \mid 1) &= 2\ln\big(2 + x_i(0) + e_i(1) - e_i(0)\big) = 2\ln(2 + \tfrac{2}{3}a_i + c_i) \\ &> 2\ln(2 + \tfrac{4}{3}a_i) = 2\ln\big(2 + x_i(1)\big) = U_i(x_i, 1 \mid 1). \end{aligned}$$

We also have $e_i(1) + x_i(0) - e_i(0) = \frac{2}{3}a_i + c_i > \frac{4}{3}a_i \geq 0$; that is, the net trade $x_i(0) - e_i(0)$ is feasible.

Since c induces the uniform distribution on $[0, 1]$, we have $\lambda(B \cup C) = 1$. Hence, the allocation x leads to almost every agent $i \in I$ to have the incentive to misreport her type. □

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